Synergistic fibre strengthening in hybrid composites

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The theoretical stress–strain behaviour of a three component hybrid composite consisting of a brittle fibre and a "matrix" composed of a binder and less brittle fibre is described, and the conditions required for a synergistic strengthening of the brittle fibres are compared for the limiting assumptions of a frictional and elastic bond. The theory is tested using a dispersed type I carbon–glass–epoxy hybrid and it is shown that increases in effective fibre strength of around 100% can be obtained. Finally, the economic and structural advantages of using a hybrid in place of a conventional carbon composite of the same modulus are discussed.

1. Introduction

It is now well established that the matrix cracking strain of a brittle matrix composite can, under certain conditions, be raised above its normal value and there are numerous examples of this effect with fibre-reinforced cement [1–3]. The conditions to be met are that the volume fraction of fibres, V_f, must be sufficient to withstand the additional load when the matrix cracks, i.e. $V_f > V_{crit}$ and their diameter must be below a critical value which can be calculated from the other parameters of the system. The analogous case of a brittle fibre in a high strain metal or plastics matrix has received less attention because the corresponding condition for fibre strenghthening, i.e. that the matrix must bear the load when the fibres break, or $V_f < V_{crit}$ means that the composite will be weaker than the unreinforced matrix and so of not much practical value. However, the critical design parameter of a structure is often its stiffness rather than strength and if the "matrix" is itself a composite of adequate strength such as grp, there remains the possibility of maintaining the increased stiffness obtained by adding a high modulus fibre such as carbon, to greater composite strains than the ultimate strain of the carbon alone – in other words, a synergistic strengthening of the graphite is obtained. Moreover, when the carbon does eventually fail, it should do so in a controlled manner giving rise to a smooth inflexion in a continuously rising stress–strain curve and so contribute to a fail-safe design.

When contained in a composite material the brittle fibres will only break at the failure strain of the fibres alone if sufficient work of fracture is available to form the new fibre surfaces. When failure of the fibres leads to immediate failure of the whole composite this condition is obviously satisfied. However, if the composite remains intact at the normal failure strain, the potential deformation upon fibre failure may provide insufficient energy for the failure to proceed. In this case, the fibres will not fail but will remain intact until a strain high enough to produce the required work of fracture is reached.

In this paper we estimate the failure strain of the more brittle (unidirectional) fibre in a hybrid composite on the assumption of a perfect bond between the fibre and matrix, then compare it with that predicted on the other extreme assumption of a sliding frictional bond and finally present a preliminary test of the theory using a hybrid glass–carbon–epoxy composite.

2. Theory

2.1. Condition for multiple fracture

For multiple fracture of the more brittle fibres, the load per unit area of composite at the failure strain of the fibres in the "matrix", ϵ_{fuc} (which may be equal or greater than their normal failure strain ϵ_{fu}), must be less than the strength of the "matrix" times its volume fraction, i.e.

$$E_c \epsilon_{fuc} \leqslant \sigma_{mu} V_m,$$

which, for a unidirectional composite, means (to a good approximation)

$$\epsilon_{fuc} E_f V_f + \epsilon_{fuc} E_m V_m \leqslant \epsilon_{mu} E_m V_m,$$

or

$$\alpha \geqslant \frac{\epsilon_{fuc}}{\epsilon_{mu} - \epsilon_{fuc}}, \quad (1)$$

where $\alpha = E_m V_m / E_f V_f$, E is Young's modulus, and subscripts f and m refer to the fibre and matrix respectively, where the matrix is defined as the binder plus the less-brittle fibre. For example a glass–carbon–epoxy composite with $\epsilon_{mu} \sim 0.025$, $\epsilon_{fuc} = \epsilon_{fu} \simeq 0.005$, $E_f = 300\,\text{GN m}^{-2}$ and $V_f = 0.2$ we find $E_m \geqslant 19\,\text{GN m}^{-2}$ which implies about 24% of the matrix or 20% of the total volume should be glass in order to avoid catastrophic failure when the carbon fails.

2.2. Fibre strengthening: bonded case

This is shown schematically in Fig. 1. It is assumed that wherever a fibre failure occurs, this is immediately followed by failure of all the other fibres and the failures are co-planar and perpendicular to the fibres. As the load per unit area of composite must be constant at any cross-section,

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$

or

$$\sigma_m = \frac{\sigma_c - \sigma_f V_f}{V_m}. \quad (2)$$

When the fibre breaks at a fibre stress σ_{fuc},

$$\sigma_c = E_c \epsilon_{fuc} = \sigma_{fuc} V_f + \epsilon_{fuc} E_m V_m, \quad (3)$$

and hence from Equations 2 and 3,

$$\sigma_m = \frac{V_f}{V_m}(\sigma_{fuc} - \sigma_f) + \epsilon_{fuc} E_m, \quad (4)$$

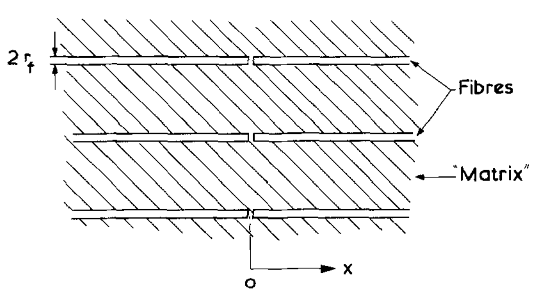

Figure 1 Hybrid composite failure model.

where σ_m and σ_f are functions of x. The exact form of the build-up in fibre stress from the borken end has been studied with essentially similar results by a number of workers [4–6] following the original shear lag analysis of Cox [7]. In the present paper we use the form derived by Rosen [4]

$$\sigma_f = \sigma_{fuc}(1 - e^{-\eta x}), \quad (5)$$

where

$$\eta^2 = \frac{G_m}{E_f} \cdot \frac{V_f^{1/2}}{1 - V_f^{1/2}} \left(\frac{1}{r_f}\right)^2,$$

G_m is the shear modulus of the matrix, and r_f the fibre radius. From Equations 4 and 5,

$$\sigma_m = \Delta\sigma_{mo} e^{-\eta x} + \epsilon_{fuc} E_m$$

where $\Delta\sigma_{mo} = \sigma_{fuc} V_f / V_m$ is the additional stress on the matrix at $x = 0$ when the fibres break. The additional matrix stress at distance x from the broken end is, therefore,

$$\Delta\sigma_m = \Delta\sigma_{mo} e^{-\eta x}$$

and the additional extension of the matrix (equal to the extension of the composite) as a result of this additional stress (for two sides of the break) is

$$\delta l = \frac{2\Delta\sigma_{mo}}{E_m} \int_0^\infty e^{-\eta x} dx = \frac{2\epsilon_{fuc}}{\eta\alpha}. \quad (6)$$

The work done by the applied stress per unit area of cross-section is $\frac{1}{2} E_c \epsilon_{fuc} \delta l$ and so if γ_f is the surface work of fracture of the fibre, it cannot break unless

$$\frac{E_c \epsilon_{fuc}^2}{\eta\alpha} \geqslant 2\gamma_f V_f, \quad (7)$$

or in the limiting case

$$\epsilon_{fuc}^2 = \frac{2\gamma_f}{E_f(1 + 1/\alpha) r_f} \cdot \left(\frac{G_m}{E_f}\right)^{1/2} \frac{V_f^{1/4}}{(1 - V_f^{1/2})^{1/2}}. \quad (8)$$

Equation 8 defines the minimum fibre strain, ϵ_{fuc}, at which sufficient work of fracture will be available to follow the fibres to fail. For a given composite system this strain will vary inversely with the square root of the fibre radius.

2.3. Fibre strengthening: debonded case

This has already been considered by Aveston *et al.* [8]. A basic assumption of this theory is that the shear strength τ has a constant limiting value. Dividing their Equation 26 by E_f convert to a strain we get

$$\epsilon_{\mathrm{fuc}}^3 = \frac{12\gamma_{\mathrm{f}}\tau E_{\mathrm{m}} V_{\mathrm{m}}^2}{E_{\mathrm{c}} V_{\mathrm{f}} E_{\mathrm{f}}^2 r_{\mathrm{f}}}, \tag{9}$$

which is symmetrical with their Equation 21 for the cracking strain of a brittle matrix. In general, the value of the frictional shear strength of the interface does not easily lend itself to direct measurement and there is often some doubt about the value of r_{f} when the fibres are in a semi-dispersed bundle. We can overcome both these problems by measuring the transfer or ineffective length x', i.e. the distance from the crack needed for the stress in the fibre (or matrix) to build up again to its original value. By equating the total shear and tensile forces in the fibre

$$2\pi r_{\mathrm{f}}\tau x' = \pi r_{\mathrm{f}}^2 \sigma_{\mathrm{fuc}},$$

we obtain

$$x' = \frac{\sigma_{\mathrm{fuc}} r_{\mathrm{f}}}{2\tau} = \frac{\epsilon_{\mathrm{fuc}} E_{\mathrm{f}} r_{\mathrm{f}}}{2\tau} \tag{10}$$

which on substitution into Equation 9 gives

$$\epsilon_{\mathrm{fuc}}^2 = \frac{6\gamma_{\mathrm{f}} E_{\mathrm{m}} V_{\mathrm{m}}^2}{x' E_{\mathrm{c}} V_{\mathrm{f}} E_{\mathrm{f}}} = \frac{6\gamma_{\mathrm{f}} \alpha V_{\mathrm{m}}}{x' E_{\mathrm{c}}}. \tag{11}$$

The theories for the elastic and debonded cases may be compared by combining Equations 8 and 11 to give

$$\frac{\epsilon_{\mathrm{fuc}}^2}{\epsilon_{\mathrm{fuc}}^2(\text{debonded})} = \frac{x'}{3r_{\mathrm{f}}}\left(\frac{G_{\mathrm{m}}}{E_{\mathrm{f}}}\right)^{1/2} \frac{V_{\mathrm{f}}^{1/4}}{(1-V_{\mathrm{f}}^{1/2})^{1/2}} \cdot \frac{V_{\mathrm{f}}}{V_{\mathrm{m}}}. \tag{12}$$

As an upper limit to τ we can put it equal to the shear strength of the matrix τ_{u} and substitute into Equation 10 to give

$$\frac{x'}{r_{\mathrm{f}}} = \frac{\sigma_{\mathrm{fuc}}}{2\tau_{\mathrm{u}}} = \frac{\beta}{2} \cdot \frac{E_{\mathrm{f}}}{G_{\mathrm{m}}}. \tag{13}$$

where β is the ratio of the fibre failure strain in tension to the matrix failure strain in shear, so that Equation 12 becomes

$$\frac{\epsilon_{\mathrm{fuc}}^2}{\epsilon_{\mathrm{fuc}}^2(\text{debonded})} = \frac{\beta}{6}\left(\frac{E_{\mathrm{f}}}{G_{\mathrm{m}}}\right)^{1/2} \frac{V_{\mathrm{f}}^{1/4}}{(1-V_{\mathrm{f}}^{1/2})^{1/2}} \cdot \frac{V_{\mathrm{f}}}{V_{\mathrm{m}}}. \tag{14}$$

As an example, taking values of $V_{\mathrm{f}} = 0.03$, $E_{\mathrm{f}} = 300$, $E_{\mathrm{m}} = 20$ appropriate to the experiments in the next section, and putting

$$G_{\mathrm{m}} = 2G_{\mathrm{resin}} = E_{\mathrm{resin}} = 3\,\mathrm{GN\,m^{-2}}$$

[9] we obtain

$$\frac{\epsilon_{\mathrm{fuc}}^2}{\epsilon_{\mathrm{fuc}}^2(\text{debonded})} = 0.024\beta.$$

As β is likely to be somewhat less than unity we see that for a strong frictional bond with $\tau = \tau_{\mathrm{u}}$ the debonded condition should lead to a higher effective fibre strength. As V_{f} is increased the fibre failure strains predicted by the two theories become closer and at $V_{\mathrm{f}} = 0.2$ for $\beta = 1$ and $\tau = \tau_{\mathrm{u}}/2$ are about the same.

When the fibres fail at a strain ϵ_{fuc}, then by analogy with the brittle matrix case [8] the additional stress in the matrix will vary between $\sigma_{\mathrm{mu}} V_{\mathrm{f}}/V_{\mathrm{m}}$ at the plane of the fibre breaks and zero at distance x' from the break so that the average additional strain as a result of multiple cracking of the fibres is $\epsilon_{\mathrm{fuc}}/2\alpha$. When multiple fracture of the fibres is complete the matrix

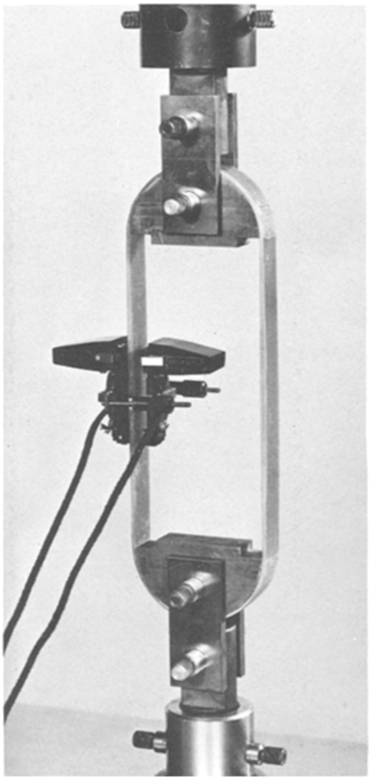

Figure 2 Test rig showing modified NOL ring specimen.

bears any additional load and so the Young's modulus becomes $E_m V_m$.

3. Experimental

For maximum effect the brittle fibres should be as thin as possible (see Equations 8 and 9), discrete and distributed evenly throughout the matrix. This is virtually impossible to achieve with continuous fibres and the nearest approach that could be made was to use a carbon–glass–epoxy system composed of alternate layers of 204-filament glass strands and 10 000 filament type I (Courtaulds) carbon fibre which had been expanded to form a semi-aligned veil about 15 cm wide. Specimens were prepared by filament winding onto either rectangular or modified NOL type formers (Fig. 2) each layer being impregnated with Ciba resin MY753 containing 10% by weight of hardener HY951, and cured at room temperature to avoid complications from differential contractions of the two fibres. Strains were measured over a 5 cm gauge length on each side of the specimen using a special Instron averaging extensometer which drove the chart recorder. The cross-head speed was 1 mm min^{-1} for the modified NOL ring and straight specimens (~ 20 cm between grips) and 0.5 mm min^{-1} for dog bone specimens where the reduced section was 75 mm.

4. Results and discussion

4.1. Test of theories

A typical stress–strain curve for a hybrid containing 3.5 vol % type I carbon and 35% glass is shown by the full line OC′ in Fig. 3. Also shown is the curve OABC predicted by the debonded theory and the observed failure strain of the "matrix" ϵ_{mu} and type I carbon–epoxy composites ϵ_{HM} tested alone. The essential features of the theory – an initial slope of $E_f V_f + E_m V_m$, a final slope of $E_m V_m$ displaced by a strain $\epsilon_{fuc}/2\alpha$ and a mean fibre breaking strain ϵ_{fuc} much greater than that of the plain carbon composite ϵ_{HM} – are all reflected in the experimental stress–strain curve of the hybrid. Although no cracks as such could be seen and the surface remained undamaged, a regular pattern of white striations corresponding to a debonded interface at breaks in the carbon fibres became just visible to the naked eye as the specimens were strained passed the inflexion point. Their spacing, averaged over a number of regions of the specimen was 1.0 mm (Fig. 4).

The one significant difference between theory and experiment is that the fibres break over a stress range instead of the single value given by the line AB with the result that the initial departure from linearity corresponding to the initial fibre breaks is difficult to define. A similar problem

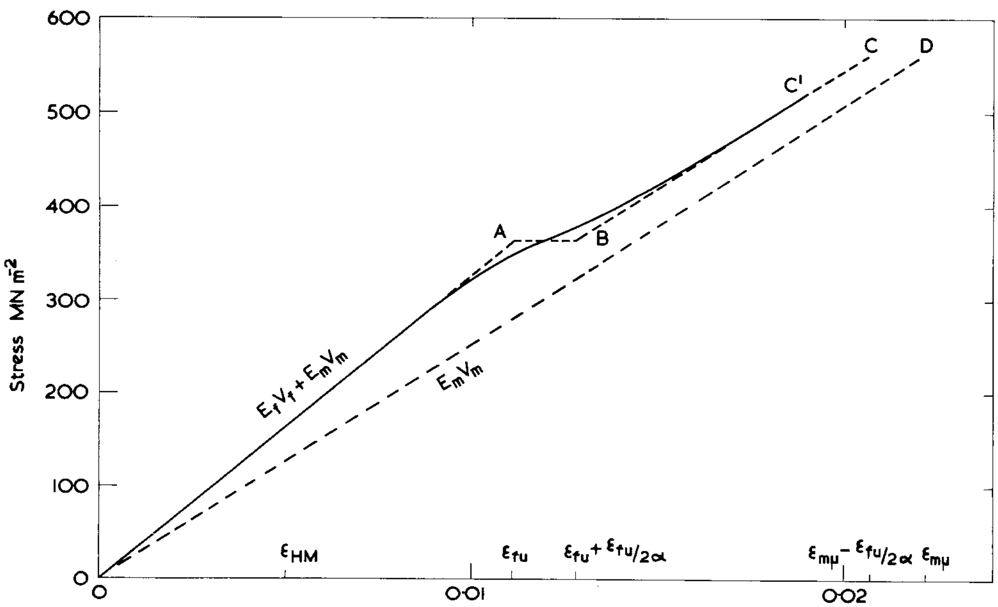

Figure 3 Tensile stress–strain diagram for hybrid carbon–glass–epoxy composite. Full line is the experimental curve and broken line OABC is predicted by the debonded theory.

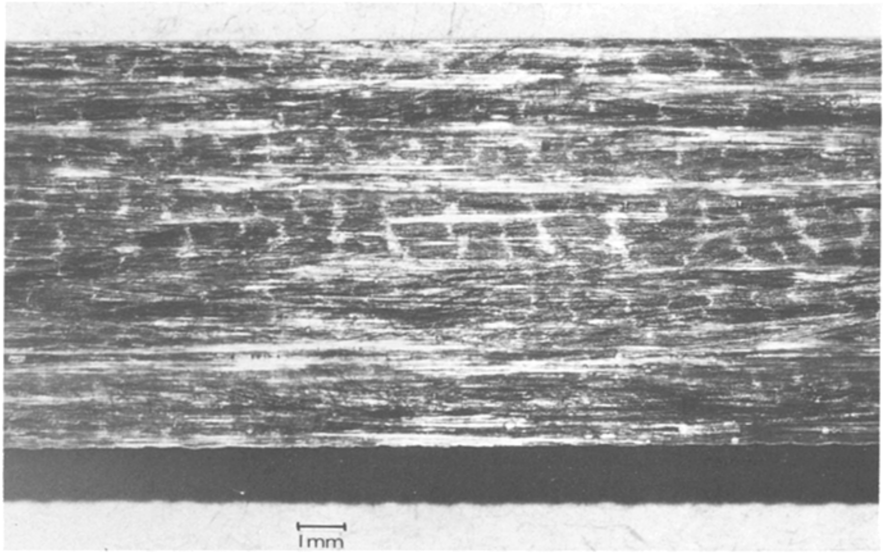

Figure 4 Hybrid composite after testing showing debonded areas (vertical white regions spaced at ~ 1.0 mm) at the end of broken fibres.

arose with the brittle matrix composites [1] and as in that case we take the stress at the minimum slope of the stress strain curve as the mean cracking stress and draw the horizontal line AB of length $\epsilon_{fuc}/2\alpha$ at this point, and define the corresponding elastic strain at A as the mean fibre cracking strain ϵ_{fuc}. The acoustic emission was also a maximum at this point.

The mean fibre breaking strain so defined, averaged over six specimens was $1.08 \pm 0.02\%$ (one standard deviation of the points) and the mean limit of proportionality $0.91 \pm 0.04\%$. These may be compared with an overall mean strain to failure (all specimen shapes) for the plain type I carbon composites of $0.50 \pm 0.06\%$. Rather surprisingly the modified NOL ring configuration designed to overcome grip alignment problems and associated stress concentrations tended to give low failure strains (0.46 ± 0.05) than the straight (0.48 ± 0.04) or dog bone (0.56 ± 0.02) specimens. The highest single value observed for any of the specimens was 0.57%. Thus although there is the usual uncertainty in measuring the true failure strain of a material as brittle as treated type I cfrp, a separation of the order of ten standard deviations between the failure strain of the carbon fibre in the hybrid and in the plain resin leaves little doubt that a real increase in effective fibre strength has been obtained.

In order to quantitatively establish whether the increase in failure strain is consistent with the predictions of the elastic or debonded theories given by Equations 8 and 11 we need to know γ_f, the surface work of fracture of type I carbon fibre. As far as we know this has not been measured, and indeed it is difficult to conceive of any method by which it could be. However, if we substitute into Equation 11 the value for graphite of $150\,J\,m^{-2}$ measured by Davidge and Tappin [10], together with the observed value of 1 mm for the crack spacing (remembering that the crack spacing will lie between x and $2x$ with a most probable value of $1.364x$ [1] we get a theoretical cracking strain of 1.10% – an agreement with experiment that seems almost too good to be true.

For the elastic theory we find an upper bound of 0.39% corresponding to discrete carbon fibres, using Equation 8 and putting r_f equal to $4\,\mu m$. In practice, owing to bunching of the fibres the effective radius will be greater and hence the predicted strain even less than this value, and so the elastic theory is inapplicable. The evidence of a debonded region in Fig. 3, the residual strain that is observed after unloading a hybrid that has been strained past the inflexion in the stress–strain curve, and the various estimates of the shear stress at the end of a broken fibre [11] all lead to the same conclusion and thus, as with the brittle matrix case, it is the debonded theory that should be used.

Comparison of the above results with previous investigations of hybrid composites – for example by Bunsell and Harris [12], Hancox and Wells [13], and Dukes and Griffiths [14] – is not

TABLE I

Composite composition	Fibre properties		Cost	
	E_f ($GN\,m^{-2}$)	σ_{fu} ($GN\,m^{-2}$)	Fibre (£ kg^{-1})	Composite (£ $litre^{-1}$)
Grafil Hm ($V_f = 0.10$) + glass ($V_f = 0.50$)	–	–	–	16.5
Grafil A ($V_f = 0.38$)	175	2.3	52	35.5
Grafil HT ($V_f = 0.30$)	220	2.6	70	37.8
Grafil HM ($V_f = 0.21$)	310	2.1	85	32.1

entirely relevant, as in each case the hybrid was composed of relatively massive layers of grp and cfrp, and as we should expect, little or no synergistic effect was observed. Also, the rather low failure strain for cfrp of 0.3% reported by Bunsell and Harris could suggest that some problems with specimen alignment remained.

4.2. Practical implications

Although the observed and calculated fibre breaking strains are in good agreement, an alternative explanation for the enhanced breaking strain is simply that the catastrophic crack propagation that results from failure of a few fibres, or possibly a surface flaw in a brittle type I cfrp, is prevented by the intervening glass fibres. A proper test of the theory would require a value of γ_F, and hybrids with a range of volume fractions of carbon fibres disposed more uniformly throughout the composite to test the dependence of ϵ_{fuc} on E_m, V_m etc, predicted by Equation 9. The fibre alignment process developed by the Explosives Research and Development Establishment [15] provides a unique method of producing such a material and a start has been made with ERDE on further testing of the theory using hybrids produced by this method.

However, the exact mechanism does not affect the fact that it whould be possible to produce a hybrid that under certain conditions is both better *and* at the same time cheaper than normal cfrp. Thus a frequent design requirement is for a material that is appreciably stiffer than grp yet retaining much of its inherent fracture

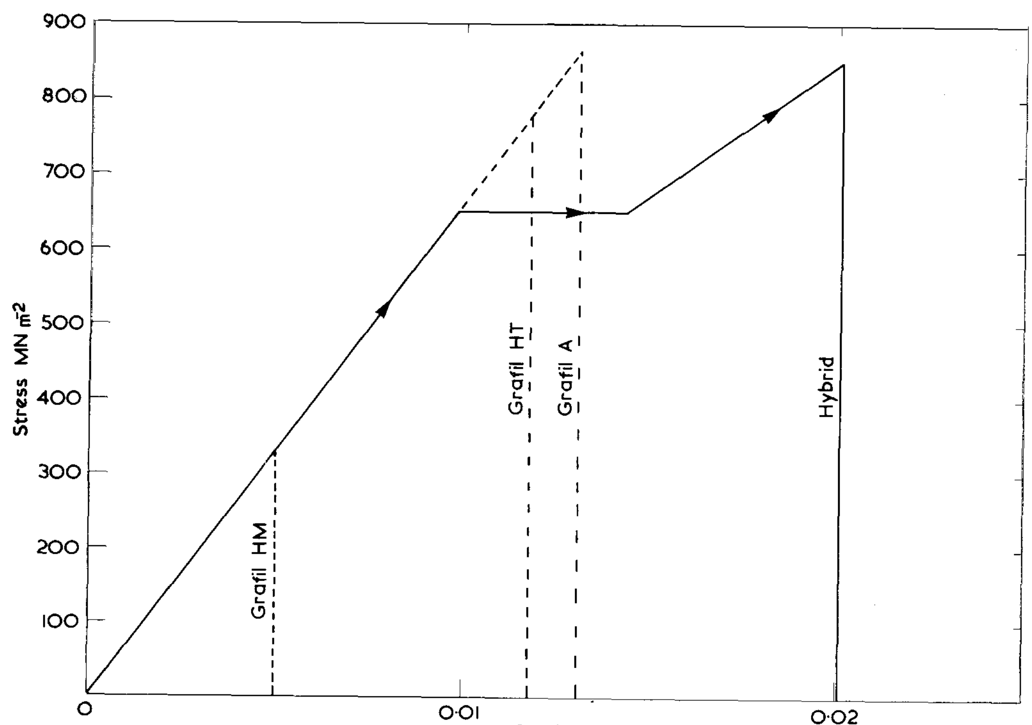

Figure 5 Stress–strain curves predicted for the four composites with $E = 66\ GN\,m^{-2}$ listed in Table I.

toughness. For example, suppose a Young's modulus of 66 GN m^{-2} ($\sim 10^7$ psi) is required to be maintained up to 50% of the UTS of the composite and thereafter the strain capacity is to be a maximum to provide the best safety factor under impact conditions or local overstrain – in other words, the kind of intermediate application for which the cheaper grades of carbon such as Courtaulds Grafil A were developed. The initial modulus requirement could be met by any of the four materials listed in Table I but the cost of the hybrid (1975 prices for 50 kg quantities) would be about half that of the alternatives. Moreover, the corresponding stress–strain curves (using the debonded theory for the hybrid and the manufacturers figures for the ratios of the minimum fibre UTS to Young's modulus for the ultimate strains of the Grafil A and Grafil HT (type II carbon) composites) show the total energy to failure of the hybrid to be of the order of twice the best of the alternatives.

5. Conclusion

Just as the failure strain of a brittle matrix may be increased by reinforcement with thin stiff fibres as long as these remain intact when the matrix has cracked, so in theory can the failure strain of brittle fibres in a matrix that remains intact at the normal fibre failure strain. For the brittle matrix case the failure strain is predicted to be higher for an elastic fibre–matrix bond, whereas for brittle fibres a sliding frictional bond leads to a higher value. In practice debonding will always occur so that the constrained failure strains of brittle fibres are an order of magnitude greater than that of brittle matrices. A preliminary test of the theory using a type I carbon–glass–epoxy hybrid composite confirmed that the failure strain of the carbon fibres can be increased to a value of about 1%.

Acknowledgements

We are grateful to Dr A. Kelly and Dr F. J. Lockett for helpful comments on the manuscript.

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Received 11 February and accepted 17 March 1976.